FISCAL IMBALANCE IN AN ECONOMY WITH INFORMALITY:

Evidence from Dynamic Stochastic General Equilibrium Model

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ABSTRACT

Efforts to pull the Nigerian economy out of recession have compelled the use of expansionary fiscal policy. This paper simulates a fiscal policy shock in a New Keynesian DSGE model with varying sizes of informality. The model incorporates price stickiness in the formal sector, while some degree of price flexibility, non-Ricardian households and untaxed credit constraint are introduced in the informal sector. In addition, government has nominal budget constraints; therefore, deficits are financed through borrowing. The paper finds that the policy has a stabilizing-effect on aggregate demand with relatively more informal goods consumption; the untaxed informal sector's goods leaks government revenue. Conversely, there is relatively more output growth in formal firms. Informality distorts short-run aggregate investment but such effect vanishes over time. The paper concludes that, though expansionary fiscal policy stabilizes aggregate demand during an economic downturn, however, the presence of a large informal sector distorts fiscal revenue and creates fiscal imbalance.

Key words: fiscal policy, stabilization, general equilibrium, modelling, informal economy

JEL classification: H30, H31, H32, H62

1. Introduction

The efforts to pull the Nigerian economy out of recession, which began in the third quarter of 2014, appear to have compelled expansionary fiscal policy

over the last few years. Government seems to sustain its spending on both current and capital expenditure probably with the view to stimulate aggregate demand in the economy. Perhaps constrained by a decline in oil revenue[1] during the period, with fiscal revenue also dwindling, both domestic and foreign debt appear to be rising. Non-oil revenue tax, which could be seen as a major source of revenue to augment the fall in oil revenue, could offer a viable revenue option for government. This means that revenue from value added tax (VAT), sales profit tax, companies gained tax, and other government sources could become necessary to improve the fiscal revenue side or, at best, reduce possible fiscal imbalance and reduce government deficits.

In view of the possible multiplier effect of an expansionary fiscal policy, the response of aggregate demand to a fiscal stimulus may not only depend on the size of government spending, but possibly, also on the structure of the economy. Specifically, the relative size of the formal-to-informal sector could either constrain or propagate the effect of the fiscal stimulus, and to a greater extent, might distort the fiscal balance.[2] Multilateral institutions such as the International Monetary Fund (IMF) report that the size of the informal sector in developing countries is relatively larger than in developed economies. Similarly, it is believed that the relative size of the informal sector is disproportionately larger than the formal economy. For instance, the IMF (2017) annual report indicated that 65% of Nigeria's GDP was accounted for by the informal sector. This suggests that the response of the entire economy to fiscal stimulus would depend largely on the efficiency of the informal sector. In addition, the size of revenue income as well as long-term deficit might be unsustainable due to leakages in the informal sector. Therefore, the dynamics of the size of the informal economy could exert some potential distortions that might create fiscal imbalance.

Dynamics in the relative size of informality in an economy could have a far-reaching effect on the potency as well as the sustainability of the expansionary fiscal policy embarked upon. Two contrasting debates have already emerged explaining the relative significance and role of the informal

[1] Due to negative oil price shock.

[2] This is the equilibrium between revenue income from tax and public expenditure.

sector on the economy. Early arguments put forward by scholars such as Leal (2014), Maloney & Saavedra-Chanduvi (2007), Schneider (2006), Frey & Schneider (2000) and Schneider & Enste (2000) indicate that the informal sector is relatively more efficient than the formal sector. Therefore, expansionary effect of fiscal policy is more potent in the informal economy and by extension the efficacy of the policy on the economy will be greater if the degree of informality is higher (larger relative size of informal sector). However, recent evidence forwarded in studies such as Batini et al. (2010; 2011), Senbeta (2013), Bandaogo (2016) and Dellas et al. (2017) among others, observed that the informal sector is inefficient and poses a distortionary effect on fiscal policy. It is in light of this ongoing debate and the recent fiscal policy stands in Nigeria that the following questions arise; what is the possible effect of the fiscal expansion on aggregate demand; what is the relative effect of the policy on output and consumption across formal and informal sectors; what is the effect of informal sector dynamics on the potency of the fiscal stimulus; what is the inflationary effect of the fiscal stimulus?

To address these questions, this paper developed a New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model for the Nigerian economy, in the spirit of Gali and Monacelli (2005), Batini et al. (2011), Senbeta (2013), Haider et al. (2014). In line with the micro foundation of the baseline NK- DSGE, key and unique features found in the Nigeria economy were introduced, one of which is the large and dynamic size of the informal sector. Other features introduced in the informal economy include non-Ricardian households and untaxed credit constraint firms that produce tradable and non-tradable goods. In stark contrast to the informal sector, the model incorporates price stickiness in the formal sector. The government is assumed to be faced with nominal budget constraints; therefore, deficits are financed through public borrowing.

The key message that emerged from this paper is that the size of informality and the changes in its relative size substantially influence the potency of an expansionary fiscal policy. Furthermore, informality has the propensity to distort the revenue side of fiscal policy. In large part, the distortion arises due to leakages in untaxed informal intermediate and final goods consumption. The magnitude of the revenue leakage and the associated

fiscal expenditure create a fiscal imbalance. The broad implication of this key finding is that, a disproportionately larger rise in informal goods consumption relative to output growth of the informal goods-producing firms further magnifies the fiscal imbalance which makes deficit financing persistent and keeps public debt at unsustainable levels.

The rest of the paper is structured as follows. Section 2 provides a review of some of the empirical literature. In section 3, the NK-DSGE model is presented, while section 4 is the analysis and discussion of the simulation results. Section 5 concludes the paper.

2. Literature Review

In response to the increasing debate on the applicability of the DSGE model in a developing economy, much improvement is desired in the model to reflect important elements of these economies (see Blanchard & Gali, 2010; Senbeta, 2011; Batini et al., 2010; 2011). Moreover, the realization of the benefit of internalizing the informal economy into a macroeconomic model, especially for a developing economy with a large shadow economy, cannot be overemphasized. The evidence that attests to this fact is consistent with Dellas et al. (2017). They argued that "any model that omits the informal economy can systematically miss-forecast and miss-quantify the effect of fiscal policy." This view is verified and consistent with Senbeta (2011; 2013) and Bandaogo (2016). This argument provides an important justification why it is imperative for the application of the DSGE in countries with large informal economies to incorporate such feature.

Meanwhile, a contrasting and ongoing debate exists on the role of informality in the economy. On one hand, is the reason put forward by Leal (2014) that there is evidence of several margins of distortions (relating to productivity and output) in the economy, which are consequences of informality. The study provided the evidence of a U-shaped trade-off between aggregate output and size of the informal economy. On the other hand, is the contention that informality constitutes an efficient portion of the economy and therefore it is beneficial to the aggregate economy. The belief within this strand of literature is that the informal economy is resilient to shocks. The

proponents of this argument include Schneider and Enste (2000), Schneider (2005), Maloney and Saavedra-Chanduvi (2007) and Schneider (2006).

Thus, divergence and the contestation on the role of informality between the proponents and the anti-proponents can be attributed to the differences in the methodological approaches used by the two contesting streams of literature. The partial equilibrium approach used by the proponents is in stark contrast to the general equilibrium approach. However, it is believed that the general equilibrium approach provides a far superior basis than the partial equilibrium. One strong reason for the better performance of the general equilibrium methodology is the encompassing framework that illuminates the linkages of the informal economy with other economic agents in other non-informal sectors and institutions. Consequently, economic dynamics and business cycles do not occur partially (based on isolated sectors) rather fluctuations occur in the general economy. Therefore, it is pertinent to ascribe the better judgment of the true reflection of the informality within the general equilibrium. Henceforth, if this fair inference is to be held as a fact, then it is valid to assume the shadow economy as a source of shock in an economy.

3. Model

The theoretical foundation of the SOE model is in the spirit of Gali and Monacelli (2005), Blanchard and Gali (2010) and Bergholt (2014). The justification for rooting the model in this canonical setup is to allow for the model to mimic real world settings, with an open economy which has an external sector and trade with the rest of the world. In order to incorporate the multi-sector economy that bears heterogeneity with a dual economy that is made up of formal and informal economic agents, consisting of households and firms, the SOE model will follow Batini et al. (2011), Senbeta (2013), Haider et al. (2012), and Bandaogo (2016; 2018). Firms are assumed to set prices in a Calvo staggered[3] manner.

[3] This is consistent with the Calvo (1983) formulation of price setting behaviour of firms.

3.1 Household

The household sector is captured by an Euler equation which shows the utility derived from intertemporal optimization of total household consumption (C_T), fraction of household labour supply to the informal economy ($N^{s}_{\alpha i,t}$), share of household labour hours supply to the formal economy ($N^{f}_{\alpha i,t}$), real money balances held by the household ($\frac{M_i(k)}{P_i}$) at any given time, and fraction of leisure hours used by the household ($1-\ell_i(k)$).

$$E\sum_{t=i}^{\infty}\beta^t\, u_t\left\{C_T; N^{s}_{\alpha i,t}; N^{f}_{\alpha i,t}; \frac{M_i(k)}{P_i}; 1-\ell_i(k)\right\}$$

$$u_t\left\{C_T; N^{S}_{\alpha i,t}; N^{F}_{\alpha i,t}; \frac{M_i(k)}{P_i}; 1-\ell_i(k)\right\} =$$

$$= \xi_{c,t}\left[\ln(C_T(k)-\psi C_{T-1}) - \int \frac{\left(N^{s}_{\alpha i,t}\right)^{1+\psi}}{1+\psi}\, d\alpha_i \right.$$

$$\left. - \int \frac{\left(N^{f}_{\alpha i,t}\right)^{1+\psi}}{1+\psi}\, d\alpha_i \frac{\xi_m}{\mathbb{Q}}\left(\frac{M_i(k)}{P_i}\right)^{\mathbb{Q}} - \xi_{\ell,t}\frac{\ell_i(k)^{1+\sigma_L}}{1+\sigma_L}\right]$$

where $\xi_{c,t}$ is the household's preference shock which accounts for inverse elasticity of intertemporal consumption substitution.

The aggregate household consumption ($C_T(k)$) is a sum of the formal sector's goods consumption ($C_t^{\frac{(\rho_1-1)}{\rho_1}}$) and the informal sector's goods consumption ($C_n^{\frac{(\rho_1-1)}{\rho_1}}$). The aggregation of the consumption is presented below:

$$C_T(k) = \left[(1-\vartheta_1)^{\frac{1}{\rho_1}} C_t^{\frac{(\rho_1-1)}{\rho_1}} + \vartheta_1^{\frac{1}{\rho_1}} C_n^{\frac{(\rho_1-1)}{\rho_1}}\right]^{\frac{\rho_1}{(\rho_1-1)}}$$

3.1.1 *Households' Optimal Consumption Behaviour*

The respective consumption of formal and informal sectors goods is presented below. It is noteworthy that consumption of each sector's goods is represented as a fraction of the aggregate consumption.

Consumption formal sector's good as a fraction of $C_T(k)$:

$$C_{ti}(k) = (1-\vartheta_1)\left(\frac{p_{ti}}{P_T}\right)^{-\rho_1} C_T(k)$$

Consumption informal sector's good as a fraction of $C_T(k)$:

$$C_{ni}(k) = \vartheta_1\left(\frac{p_{ni}}{P_T}\right)^{-\rho_1} C_T(k)$$

Therefore, a composite consumer price index which constitutes tradable and non-tradable commodities that enter the household at any given time is represented as:

$$P_T = [(1-\vartheta_1){p_{ti}}^{1-\rho_1} + \vartheta_1 {p_{ni}}^{1-\rho_1}]^{\frac{1}{1-\rho_1}}$$

$$C_{t,i}(k) = \left[(1-\vartheta_2)^{\frac{1}{\rho_2}} C_{D,i}(k)^{\frac{(\rho_2-1)}{\rho_2}} + (\vartheta_2)^{\frac{1}{\rho_2}} C_{M,i}(k)^{\frac{(\rho_2-1)}{\rho_2}}\right]^{\frac{\rho_2}{(\rho_2-1)}}$$

$$C_{M,i}(k) = (\vartheta_2)\left(\frac{p_{M,i}}{P_{t,i}}\right)^{-\rho_2} C_t(k)$$

$$C_{D,i}(k) = (1-\vartheta_2)\left(\frac{p_{D,i}}{P_{t,i}}\right)^{-\rho_2} C_t(k)$$

Tradable price index:

$$p_t = \left[(1-\vartheta_2){p_{D,i}}^{1-\rho_2} + \vartheta_2 {p_{M,i}}^{1-\rho_2}\right]^{\frac{1}{1-\rho_2}}$$

3.1.2 *Households' Optimal Labour Supply Behaviour*

The total hours of household labour supply is defined by the sum of thje fraction of household labour hours supply to the formal sector ($\ell_{f,i}(k)^{\frac{(\mu_\ell-1)}{\mu_\ell}}$) and the informal economy ($\ell_{s,i}(k)^{\frac{(\mu_\ell-1)}{\mu_\ell}}$).

$$\ell_{T,i}(k) = \left[(1-\theta_\ell)^{-\frac{1}{\mu_\ell}}\ell_{s,i}(k)^{\frac{(\mu_\ell-1)}{\mu_\ell}} + (\theta_\ell)^{-\frac{1}{\mu_\ell}}\ell_{f,i}(k)^{\frac{(\mu_\ell-1)}{\mu_\ell}}\right]^{\frac{\mu_\ell}{(\mu_\ell-1)}}$$

Fraction of household labour hours supply to informal economy:

$$\ell_{s,i} = (1-\gamma_s)\ell_{s,t-1}$$

Fraction of household labour hours supply to formal sector:

$$\ell_{f,i} = (1-\gamma_f)\ell_{f,t-1} + \tau_f$$

Unemployment at the beginning of period:

$$(1-N)_t = 1-(1-\gamma)(\ell_{f,t-1} + \ell_{s,t-1})$$

Unemployment at the end of period:

$$(1-N)_{t+i} = 1-(\ell_{f,t-1} + \ell_{s,t-1})$$

Assuming that wages in the informal sector are greater than wages in the informal economy (contributory pension and other allowances are unobtainable in the informal sector),

$$w_f > w_s$$

Given the prices of wages in the formal and shadow economy, therefore, household optimal objective function is subject to intertemporal budget constraint, determined by the price of consumption bundle and the wage income paid to labour:

$$w_f\ell_f + w_s\ell_s + d_t + r_b b_{t-1} \geq P_T C_T + b_t$$

First Order Condition FOC:

$$\xi_c(C_T(k) - \psi C_{T-1})^{-1} = \lambda_T P_T$$

$$\xi_c\xi_m\left(\frac{M_T(k)}{P_T}\right)^{\mathbb{Q}-1} = \lambda_T(k)\left(1-\beta E\frac{\lambda_{T+1}(k)}{\lambda_T(k)}\frac{P_T}{P_{T+1}}\right)$$

$$\xi_c\xi_\ell(\ell_T k)^{\sigma_\ell} = \frac{w_t\lambda_T}{P_C}$$

$$\beta E \frac{\lambda_{T+1}(k)}{P_{T+1}}$$

3.2 Formal economy production

3.2.1 *Final-Goods Producing and Exporting Firms*

Aggregate output of firms in the formal economy (q_f) is the sum of output of the individual firms in the formal sector, which is defined by:

$$q_f = \left[\int_0^1 q_f k_d^{\frac{\rho_6-1}{\rho_6}} dk_d\right]^{\frac{\rho_6}{\rho_6-1}}$$

Final good exports: a fraction of formal firms are goods-exporting firms. Thus, the share of their output that is exported to the RoW is given as:

$$q_f^* = \left[\int_0^1 q_f^* k_d^{\frac{\rho_6-1}{\rho_6}} dk_d\right]^{\frac{\rho_6}{\rho_6-1}}$$

The profit of the formal-goods producing firms is defined by the difference between the aggregate revenue and total cost:

$$\Pi_t = (1 - T_d) p_t q_d - \int_0^1 p_t\,(k_t) q_d(k_t) dk_d$$

The formal-goods exporting firms set price and profit based on domestic cost of production:

$$\Pi_t^* = (1 - T_d) p_t q_d - \int_0^1 p_t\,(k_d) q_d(k_d) dk_d$$

Final-goods foreign demand in the domestic economy is defined by the domestic demand for foreign goods ($q_f(k_d)$).

$$q_f(k_d) = \left(\frac{p_t(k_d)}{(1 - T_d) p_t}\right)^{-\rho_6} q_t$$

$$q_f^*(k_d) = \left(\frac{p_f^*(k_d)}{(1 - T_d) p_t}\right)^{-\rho_6} q_f^*$$

3.2.2 *Final-Goods Importing Firm*

This is defined by the output of the formal goods importing firms. The aggregate output of formal goods-importing firms is defined by ($q_{f,im}$):

$$q_{f,im} = \left[\int_0^1 q_{f,im} k_d^{\frac{\rho_7 - 1}{\rho_7}} dk_d\right]^{\frac{\rho_7}{\rho_7 - 1}}$$

The profit function of the goods-importing firms is given as:

$$\Pi_{im} = (1 - T_{im,d}) p_t q_d - \int_0^1 p_{im,t}(k_t) q_{im}(k_t) dk_d$$

The aggregate demand of the formal final goods-importing firms is given as ($q_{im,f}(k_d)$):

$$q_{im,f}(k_d) = \left(\frac{p_{im,t}(k_d)}{(1 - T_{im,d}) p_t}\right)^{-\rho_7} q_{im,t}$$

3.2.3 *Intermediate-Goods Producing Firm*

$$q_i(g_t) = \text{Ä}_d \left[\alpha_d^{\frac{1}{\rho_3}} h_d(k_t)^{\frac{\rho_3^{-1}}{\rho_3}} + (1 - \alpha_d)^{\frac{1}{\rho_3}} En_d(k_t)^{\frac{\rho_3^{-1}}{\rho_3}}\right]^{\frac{\rho_3}{\rho_3^{-1}}}$$

The composite of factor inputs is given by:

$$h_d(k_t) = (\xi_\ell \ell_d k_t)^{\theta_2} (V_d k_t)^{1 - \theta_2}$$

$$\frac{\xi_{\ell,t}}{\xi_{\ell,t-1}} = \zeta_\ell$$

$$\text{Ä}_d = \text{Ä}_{d_{t-1}}^{\rho_4} \exp(\xi_{\text{Ä},t})$$

$$\zeta_\ell = (1 + g_y)^{1 - \rho_5} \zeta_{\ell,t-1}^{\rho_5} \exp(\xi_{\ell,t})$$

$$max_{k_d p_t} E_t \left\{\sum_{i=0}^{\infty} \text{Z}_{t,t+1} \emptyset_d^i \frac{\epsilon_{t+1} \text{Ш} p_d(k_d) - MC_{d,t+1}}{P_{T+1}} q_{t+1}(k_t)\right\}$$

$$MC_d = \frac{w_d \ell_d(k_d) + h_t V_d(z_d) + p_{En} En_t(k_d)}{\text{Ч}_d(k_t)}$$

3.3 Domestic informal (shadow) economy production

3.3.1 *Informal Firms' Final Goods*

$$q_{\boldsymbol{s}} = \left[\int_0^1 q_s k_s^{\frac{\rho_s - 1}{\rho_s}} dk_s\right]^{\frac{\rho_s}{\rho_s - 1}}$$

Profit:

$$\Pi_s = p_{s,t} q_{s,t} - \int_0^1 p_{u,t}\left(k_{s,t}\right) q_{s,t}\left(k_{s,t}\right) dk_{s,t}$$

Demand for intermediate product:

$$q_{s,}(k_s) = \left(\frac{p_{s,t}(k_{s,t})}{p_{s,t}}\right)^{-\rho_s} q_{s,t}$$

3.3.2 *Informal Intermediate Goods Production Firm*

$$q_s(g_s) = \text{Ά}_s\left[\alpha_s^{\frac{1}{\rho_s}} h_s(k_s)^{\frac{\rho_s - 1}{\rho_s}} + (1 - \alpha_d)^{\frac{1}{\rho_s}} En_s(k_t)^{\frac{\rho_s - 1}{\rho_s}}\right]^{\frac{\rho_s}{\rho_s - 1}}$$

The composite of factor input is given by:

$$h_s(k_s) = (\xi_s \ell_s k_t)$$

Stochastic trend in the informal labour:

$$\frac{\xi_{s,t}}{\xi_{s,t-1}} = \zeta_{s,t}$$

$$\zeta_\ell = (1 + g_y)^{1-\rho_5} \zeta_{\ell,t-1} \exp(\xi_{\ell,t})$$

$$max_{k_s p_s} E_t\left\{\sum_{i=0}^{\infty} \text{Ζ}_{t,t+1} \emptyset_s^i \frac{\epsilon_{s,t+1} \coprod p_{s,t}(k_s) - MC_{s,t+1}}{P_{T+1}} q_{s,t+1}(k_s)\right\}$$

Nominal marginal cost:

$$MC_d = \frac{w_d \ell_d(k_d) + h_t V_d(z_d) + p_{En} En_t(k_d)}{\text{Ή}_d(k_t)}$$

3.4 Capital market and formal firm investment decision

Investment decision on new capital good follows CES production function:

$$I_T = \left[Ч_I^{\frac{1}{\theta_I}} (\zeta_{I,d} I_d)^{\frac{\theta_I - 1}{\theta_I}} + (1 - Ч_I)^{\frac{1}{\theta_I}} (\zeta_{I,im} I_{im})^{\frac{\theta_I - 1}{\theta_I}} \right]^{\frac{\theta_I}{\theta_I - 1}}$$

Demand for domestic investment:

$$\zeta_{I,d} I_d = Ч_I \left(\frac{p_d}{p_I} \right)^{-\theta_I} I_T$$

Demand for imported capital good:

$$\zeta_{I,im} I_{im} = Ч_I \left(\frac{p_{im}}{p_I} \right)^{-\theta_I} I_T$$

Therefore, the aggregate price index for investment in the economy is given as:

$$P_I = \left[Ч_I {P_D}^{1-\theta_I} + (1 - Ч_I) {P_D}^{1-\theta_I} \right]^{\frac{1}{1-\theta_I}}$$

The capital accumulation follows Smet and Wouters (2007) and Haider & Khan (2008), which is also consistent with the intertemporal capital accumulation law of motion as:

$$K_{D,t+1} = \zeta_t^I I_D S \left(\frac{I_{D,t+1}}{I_{D,t-1}} \right) + (1 - \delta) K_{D,t}$$

Therefore, the objective function of a formal tradeable-goods producing firm is given as:

$$Max_{K_{D,t+1} I_{D,t+1}} E_t \left[\sum_{i=0}^{\infty} E_{t,t+1} \frac{J_{t+1} K_{D,t+1} + P_{I,t+1} I_{D,t+1}}{P_{C,t+1}} \right]$$

FOC:

$$\frac{P_{I,t}}{P_{C,t}} = \frac{Q_{I,t}}{P_{C,t}} \left[S \left(\frac{I_{D,t}}{I_{D,t-1}} \right) + S' \left(\frac{I_{D,t}}{I_{D,t-1}} \right) \frac{I_{D,t}}{I_{D,t-1}} \right] \zeta_t^I$$

$$\frac{Q_{I,t}}{P_{C,t}} \left[S' \left(\frac{I_{D,t}}{I_{D,t-1}} \right) \left(\frac{I_{D,t}}{I_{D,t-1}} \right)^2 \right] \zeta_t^I$$

$$\frac{Q_{I,t}}{P_{C,t}} = E_t E_{t,t+1}\left\{\frac{J_{t+1}}{P_{C,t+1}} + (1-\delta)\frac{Q_{t+1}}{P_{C,t+1}}\right\}$$

3.5 Aggregate equilibrium

3.5.1 *Aggregate Demand and Output*

In the domestic economy, market clearing condition for good k is given by:

$$Y_{kt} = C_{Dkt} + \int_0^1 C_{Dkt}^j \, d_j$$

Demand in substitute market is given by:

$$C_{Dkt} = \left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p}$$

Hence,

$$C_{Dt} = (1-\varepsilon_B)\left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p}\left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t$$

Aggregate domestic consumption of domestically-produced goods k, in country j is given by:

$$C_{Dkt}^j = \left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} C_{Dt}^j$$

Similarly, the aggregate domestic consumption of foreign-produced goods in country j is expressed as:

$$C_{Dt}^j = \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j}\right)^{-\varepsilon_F} C_{Ft}^j$$

Because the aggregate domestic consumption is not only determined by the domestically-produced goods:

$$C_{Ft}^j = \varepsilon_B \left(\frac{P_{Dt}^j}{P_{Ft}^j}\right)^{-\varepsilon_D} C_{Ft}^j$$

but also by the imported goods. Therefore, aggregate consumption is given by:

$$C_{Dkt}^{j} = \varepsilon_B \left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^{j}}\right)^{-\varepsilon_F} \left(\frac{P_{Ft}^{j}}{P_t^{j}}\right)^{-\varepsilon_D} C_{Dt}^{j}$$

Substituting into the market clearing condition gives:

$$Y_{kt} = (1-\varepsilon_B)\left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} \left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t + \int_0^1 \varepsilon_B \left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^{j}}\right)^{-\varepsilon_F} \left(\frac{P_{Ft}^{j}}{P_t^{j}}\right)^{-\varepsilon_D} C_{Dt}^{j}\, d_j$$

$$\therefore Y_{kt} = \left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} \left[(1-\varepsilon_B)\left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^{j}}\right)^{-\varepsilon_F} \left(\frac{P_{Ft}^{j}}{P_t^{j}}\right)^{-\varepsilon_D} C_{Dt}^{j}\, d_j\right]$$

Aggregate domestic output is then given by:

$$\Rightarrow Y_t = \left(\int_0^1 Y_{kt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}}\, d_k\right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

Substituting Y_{kt} into Y_t we have:

$$Y_t = \left\{\int_0^1 \left(\left(\frac{P_{Dkt}}{P_{Dt}}\right)^{-\varepsilon_p} \left[(1-\varepsilon_B)\left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^{j}}\right)^{-\varepsilon_F} \left(\frac{P_{Ft}^{j}}{P_t^{j}}\right)^{-\varepsilon_D} C_{Dt}^{j}\, d_j\right]\right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} d_k\right\}^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$= \left\{ \int_0^1 \left(\left[\left(\frac{P_{Dkt}}{P_{Dt}} \right)^{-\varepsilon_p \frac{\varepsilon_p - 1}{\varepsilon_p}} d_k \right] \left[(1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t \right. \right. \right.$$

$$\left. \left. \left. + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j \, d_j \right] \right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \right\}^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$= \left\{ \int_0^1 \left(\left[\left(\frac{P_{Dkt}}{P_{Dt}} \right)^{1-\varepsilon_p} d_k \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \left[(1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t \right. \right. \right.$$

$$\left. \left. \left. + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j \, d_j \right] \right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \right\}^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$= \left\{ {P_{Dt}}^{\varepsilon_p} \int_0^1 \left(\int_0^1 {P_{Dkt}}^{1-\varepsilon_p} \, d_k \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \left[(1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t \right. \right.$$

$$\left. \left. + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j \, d_j \right]^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \right\}^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$= \left\{ {P_{Dt}}^{\varepsilon_p} \left[\left(\int_0^1 {P_{Dkt}}^{1-\varepsilon_p} \, d_k \right)^{\frac{1}{\varepsilon_p - 1}} \right]^{-\varepsilon_p} \left[(1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t \right. \right.$$

$$\left. \left. + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j \, d_j \right]^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \right\}^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$${P_{Dt}}^{\varepsilon_p} {P_{Dt}}^{-\varepsilon_p} \left[(1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j \, d_j \right]$$

$$Y_t = (1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 \left(\frac{P_{Dt}}{\varepsilon_{Jt} P_{Ft}^j} \right)^{-\varepsilon_F} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\varepsilon_D} C_{Dt}^j d_j$$

$$Y_t = (1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 (P_{Dt})^{-\varepsilon_F} \left(\varepsilon_{jt} \right)^{-\varepsilon_F} P_{Ft}^{j\varepsilon_F - \varepsilon_D} P_t^{j\varepsilon_D} C_{Dt}^j d_j$$

$$Y_t = (1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 P_{Dt}^{-\varepsilon_F + \varepsilon_D} \, \varepsilon_{jt}^{\varepsilon_F - \varepsilon_D} \varepsilon_{jt}^{\varepsilon_D} P_{Ft}^{j\varepsilon_F - \varepsilon_D} P_t^{j\varepsilon_D} C_{Dt}^j d_j$$

$$Y_t = (1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \int_0^1 {P_{Dt}}^{-\varepsilon_D} \left(\frac{\varepsilon_{Jt} P_{Ft}^j}{P_{Dt}} \right)^{\varepsilon_{F-\varepsilon_D}} \left(\frac{\varepsilon_{Jt} P_t^j}{P_t} \right)^{-\varepsilon_D} C_{Dt}^j d_j$$

$$= (1-\varepsilon_B) \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} C_t + \varepsilon_B \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} \int_0^1 \left(\frac{\varepsilon_{Jt} P_{Ft}^j}{P_{Dt}} \right)^{\varepsilon_{F-\varepsilon_D}} \left(\frac{\varepsilon_{Jt} P_t^j}{P_t} \right)^{-\varepsilon_D} C_{Dt}^j d_j$$

$$= \left(\frac{P_{Dt}}{P_t} \right)^{-\varepsilon_D} \left[(1-\varepsilon_B) C_t + \varepsilon_B \int_0^1 \left(\frac{\varepsilon_{Jt} P_{Ft}^j}{P_{Dt}} \right)^{\varepsilon_{F-\varepsilon_D}} \varpi_{jt}^{\varepsilon_D} C_{Dt}^j d_j \right]$$

If the effective terms of trade in country j is given by:

$$S_t^j = \frac{\varepsilon_{Jt} P_{Ft}^j}{P_{jt}}$$

Substituting in

$$= \left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} \left[(1-\varepsilon_B)C_t + \varepsilon_B \int_0^1 \left(\frac{\varepsilon_{Jt} P_{Ft}^j}{P_{Dt}}\right)^{\varepsilon_{F-\varepsilon_D}} \varpi_{jt}^{\varepsilon_{D-\frac{1}{\sigma}}} C_{Dt}^j d_j\right]$$

hence,

$$= \left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t \left[(1-\varepsilon_B) + \varepsilon_B \int_0^1 \left(\frac{\varepsilon_{Jt} P_{Ft}^j P_{jt}}{P_{Dt} P_{jt}}\right)^{\varepsilon_{F-\varepsilon_D}} \varpi_{jt}^{\varepsilon_{D-\frac{1}{\sigma}}} C_{Dt}^j d_j\right]$$

Relevance is given to degree of openness ε_B

$$= \left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D} C_t \left[(1-\varepsilon_B) + \varepsilon_B \int_0^1 \left(S_t^j S_{jt}\right)^{\varepsilon_{F-\varepsilon_D}} \varpi_{jt}^{\varepsilon_{D-\frac{1}{\sigma}}} C_{Dt}^j d_j\right]$$

The log-linearizing around a steady state gives:

$$Y_t \approx Y - \varepsilon_D \left(\frac{P_{Dt}}{P_t}\right)^{-\varepsilon_D - 1} \frac{1}{P} C_t \left[(1-\varepsilon_B) + \varepsilon_B \int_0^1 \left(S_t^j S_{jt}\right)^{\varepsilon_{F-\varepsilon_D}} \varpi_{jt}^{\varepsilon_{D-\frac{1}{\sigma}}} C_{Dt}^j d_j\right]$$

$$\Rightarrow y_t \approx c_t + \varepsilon_B \varepsilon_F s_t + \varepsilon_B \left(\varepsilon_D - \frac{1}{\sigma}\right) q_t$$

But

$$q_t = (1-\varepsilon_B) s_t$$

Therefore,

$$y_t \approx c_t + \varepsilon_B \varepsilon_F s_t + \varepsilon_B \left(\varepsilon_D - \frac{1}{\sigma}\right)(1-\varepsilon_B) s_t$$

$$= c_t + \frac{\sigma\varepsilon_B\varepsilon_F + \sigma\varepsilon_B\varepsilon_F(1-\varepsilon_B) - \varepsilon_B(1-\varepsilon_B)}{\sigma} s_t$$

$$= c_t + \frac{\varepsilon_B[\sigma\varepsilon_F + \sigma\varepsilon_F(1-\varepsilon_B) - (1-\varepsilon_B)]}{\sigma} s_t$$

$$= c_t + \frac{\varepsilon_B[\sigma\varepsilon_F + (1-\varepsilon_B) - (\sigma\varepsilon_F - 1)]}{\sigma} s_t$$

$$\Rightarrow y_t = c_t + \frac{\varepsilon_B \omega}{\sigma} s_t$$

This will apply for each specific country. Hence for the aggregate of all the countries:

$$y_t^* \equiv \int_0^1 y_t^j dj = \int_0^1 \left(c_t^j + \frac{\varepsilon_B \omega}{\sigma} s_t^j\right) dj = \int_0^1 c_t^j dj + \frac{\varepsilon_B \omega}{\sigma} \int_0^1 s_t^j dj = \int_0^1 c_t^j dj \equiv c_t^*$$

Given that $\int_0^1 s_t^j dj = 0$

substituting will give:

$$y_t = c_t^* + \frac{1-\varepsilon_B}{\sigma} s_t + \frac{\varepsilon_B \omega}{\sigma} s_t = y_t^* + \frac{1-\varepsilon_B+\varepsilon_B \omega}{\sigma}$$

$$\Rightarrow y_t = y_t^* + \frac{1}{\sigma_{\varepsilon_B}} s_t$$

To obtain the IS equation, we substitute for c_t in the Euler equation

$$y_t - \frac{\varepsilon_B \omega}{\sigma} s_t = E_t\left\{y_{t+1} - \frac{\varepsilon_B \omega}{\sigma} s_{t+1}\right\} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

Hence,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) - \frac{\varepsilon_B \omega}{\sigma} E_t \Delta s_{t+1}$$

This is only but a closed economy IS model. To incorporate the rest of the world we insert

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1} + \varepsilon_B \Delta s_t\} - \rho) - \frac{\varepsilon_B \omega}{\sigma} E_t \Delta s_{t+1}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) - \frac{\varepsilon_B (\omega - 1)}{\sigma} E_t \Delta s_{t+1}$$

$$\Rightarrow y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) - \frac{\varepsilon_B \odot}{\sigma} E_t \Delta s_{t+1}$$

If ε_F and ε_D are significantly high y_t becomes:

$$\Rightarrow y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) - \frac{\varepsilon_B \odot}{\sigma} \varepsilon_B \odot E_t \Delta y_{t+1}^*$$

This implies that the real interest rate which is consistent with the Fishers equation $i_t - E_t \pi_{t+1}$ effext on domestic output is influenced by ε_B.

3.6 Trade balance

The trade balance is defined by the net export (NX_t). The determination of the NX_t relative to steady state is conceived as:

$$NX_t = \frac{Y_t \frac{P_t}{P_{Ht}} C_t}{Y}$$

FOC around the steady state, given that P_t=$P_{Ht} = P$ and $Y_t = C_t = Y$

Therefore, a balance trade means $NX_t = 0$ implies that:

$$NX_t = \frac{Y_t - \frac{P}{P}Y}{Y}$$

$$+ \frac{1}{Y}\left[(Y_t - Y) - \frac{P}{P}(C_t - C) - \frac{1}{P}C(P_t - P) + \frac{1}{P^2}PC(P_{D,t} - P)\right]$$

$$= \frac{Y_t - Y}{Y} - \frac{C_t - C}{C} - \frac{P_t - P}{P} + \frac{P_{D,t} - P}{P}$$

$$= (y_t - y) - (c_t - c) - (p_t - p) + \left(p_{D,t} - p\right)$$

$$y_t - c_t - p_t + p_{D,t}$$

$$y_t - c_t - \varepsilon_B s_t$$

3.7 Marginal cost and inflation dynamics (supply side)

Following from the equilibrium condition in the labour market which provides that:

$$L_t = \int_0^1 L_{i,t} di$$

This follows from:

$$L_{i,t} = \frac{Y_{i,t}}{A_t}$$

This implies that:

$$L_t = \int_0^1 \frac{Y_{i,t}}{A_t} di$$

Therefore,

$$L_t = \int_0^1 \frac{Y_{i,t}}{A_t}\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon_p} di$$

$$= \frac{Y_{i,t}}{A_t}\int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon_p} di$$

The domestic inflation is consistent with the given law of motion:

$$\pi_{D,t} = \beta E_t \pi_{D,t+1} + \lambda \widehat{mc}_t^r$$

Note:

$$\lambda \sim \frac{(1-\beta\theta)(1-\theta)}{\theta} \ (deep\ papameter)$$

Also,

$$\widehat{mc}_t^r = mc_t^r - mc^r$$

This means that marginal cost will follow from:

$$mc_t^r = -v + w_t + p_{D,t} - a_t$$

$$= -v + (w_t - p_t) + (p_t - p_{D,t}) - a_t$$

$$= -v + \tau c_t + \sigma l_t + \varepsilon_B s_t - a_t$$

Aligning the marginal cost for an open economy model by incorporating RoW equilibrium:

$$= -v + \tau\left(c_t + \frac{1-\varepsilon_B}{\tau} s_t\right) + \sigma(y_t - a_t) + \varepsilon_B s_t - a_t$$

$$= -v + \tau c_t^* + (1-\varepsilon_B)s_t + \sigma(y_t - a_t) + \varepsilon_B s_t - a_t$$

$$mc_t^r = -v + \sigma y_t^* + \vartheta y_t + s_t - (1+\vartheta)a_t$$

In order to incorporate the feature of an open economy, while retaining the indirect relationship that is apparent between marginal cost and output, through labour market, we can substitute for s_t. Hence,

$$mc_t^r = -v + (\sigma_{\varepsilon_B} + \vartheta)y_t + (\sigma + \sigma_{\varepsilon_B})y_t^* + s_t - (1+\vartheta)a_t$$

Note that the terms of trade σ_{ε_B}bear a functional relationship with degree of openness elasticity of substitution between foreign goods (supplied by RoW) and domestic goods.

3.8 New Keynesian Philips curve and dynamic IS equation

The build up to the NKPC emanates from the difference between:

$$\{mc_t^r = -v + (\sigma_{\varepsilon_B} + \vartheta)y_t + (\sigma + \sigma_{\varepsilon_B})y_t^* + s_t - (1 + \vartheta)a_t\} - \{mc_t^r = -v + (\sigma_{\varepsilon_B} + \vartheta)y_t^n + (\sigma + \sigma_{\varepsilon_B})y_t^* + s_t - (1 + \vartheta)a_t\}$$

where: $y_t^n$is the natural output in the domestic economy.

This will reduce to:

$$\widehat{mc}_t^r = (\sigma_{\varepsilon_B} + \vartheta)\tilde{y}_t$$

Then we substitute into:

$$\pi_{D,t} = \beta E_t \pi_{D,t+1} + \lambda \widehat{mc}_t^r$$

⇒

$$\pi_{D,t} = \beta E_t \pi_{D,t+1} + \lambda(\sigma_{\varepsilon_B} + \vartheta)\tilde{y}_t$$

Hence,

$$\pi_{D,t} = \beta E_t \pi_{D,t+1} + F_{\varepsilon_B}\tilde{y}_t$$

3.9 Fiscal policy

The fiscal side is made up of government that relies on revenue from the export of oil in the international market and lump sum tax from domestic economic activities in the formal sector. This means that the informal economy is outside the tax net of fiscal revenue. Government deficit is financed through either or both domestic and foreign borrowing. The premises begin with the traditional province of the fiscal policy.

$$(S_t - R_t^{no} - R_t^o - e_t \Delta F_t) = DF_t$$

where: R_t^{no} is the non-oil revenue, R_t^o is the oil revenue, e_t is the exchange rate of the naira, F_t is the level of foreign debt, $S_t$is the total government spending as defined by:

$$S_t = G_t + i_t^D B_{t-1} + e_t i_t^f F_{t-1}$$

$i_t^D B_{t-1}$ is the level of domestic debt and $e_t i_t^f F_{t-1}$is the level of foreign debt

Therefore,

$$(S_t - R_t^{no} - R_t^o - e_t \Delta F_t) + NER_t = DF_t$$

To setup an explicit government budget constraint in line with Algozhina (2015) as:

$$b_t + T_t + e_t^{\frac{\in}{t}}(R_t^* - 1)OF_{t-1}^* RER_t = G_t^I + G_t^c + R_{t-1}\frac{b_{t-1}}{\pi_t}$$

where: T_t is the aggregate tax revenue, G_t^I is the government investment and G_t^c is the government consumption, $\pi_t$is the inflation rate in the economy

$$R_t^o = e_t^{\frac{\in}{t}}(R_t^* - 1)OF_{t-1}^* RER_t$$

The oil tax (T_t^o) accrued to government in form of royalties and share of government oil sector is given by:

$$T_t^o = \aleph^o P_t^{o*} Y_t^o + \text{Ц}_t^{o*}$$

This public oil tax is accumulated into the NIR as the revenue side of the fiscal policy:

$$OF_t^* = \aleph OF_{t-1}^* + T_t^o$$

Therefore, the spending side of the fiscal policy can be decomposed into government consumption expenditure and government investment expenditure. This is given by the following rules:

$$\widehat{G_t^I} = \aleph_{GI}\widehat{G_{t-1}^I} + (1 + \aleph_{GI})\left[\delta_{GI}\widehat{Y}_t - \gamma_{GI}\hat{b}_{t-1} + \gamma_{OF}^{GI}\widehat{OR}_t\right] + \varepsilon_t^{GI}$$

$$\widehat{G_t^C} = \aleph_{GC}\widehat{G_{t-1}^C} + (1 + \aleph_{GC})\left[\delta_{GC}\widehat{Y}_t - \gamma_{GC}\hat{b}_{t-1} + \gamma_{OF}^{GC}\widehat{OR}_t\right] + \varepsilon_t^{GC}$$

Therefore, the equilibrium (fiscal balance) between government revenue and expenditure is given by:

$$\widehat{T}_t + \gamma_I\widehat{OR}_t = \gamma_b\hat{b}_{t-1} + \gamma_I\hat{G}_t^I + \gamma_C\hat{G}_t^C$$

3.10 Variables block

In solving the model, the variables that form part of the "var" block are listed table 1. Each of the variables, in part, constitutes a solution to the DSGE model. Therefore, the sector specific variables only constitute a fraction of the solved aggregate variable.

Table 1: Variables Block

Variable	Notation	Definition
A^f		Formal sector capital productivity
A^s		Informal sector capital productivity
M^f	μf	Formal firms' markup
M^s	μf	informal firms' markup
w^f		Formal sector Wage price
w^s		informal sector Wage price
I		Investment
Pi	π	Inflation
Pi^f	πf	Formal firms price adjustment
Pi^s	πs	Informal firms price adjustment
C		Aggregate consumption
C^f		Consumption of formal sector goods
C^s		Consumption of informal sector goods
G		Government expenditure
Q		Aggregate output
q^f		Formal firms' output
qs		Informal firms' output

3.11 Calibration of the model

The model is calibrated based on theoretical, empirical as well as imposed parameters. The imposed parameters are based on prior information about the economy and belief about the parameters. See table 2 for the values of the calibrated parameters.

Table 2: Model Calibration and Deeper Parameter

Calibrated parameter	Symbol	Value
Steady State Inflation Rates	$\Pi = \Pi F = \Pi *$	1.02
Consumption habit	hC	0.7
Calvo prices	Ξ	0.75
Labour share	A	0.70
Hours worked	H	1/3
Preference parameter	ϱ	calibrated so H=1/3
government spending	Gy	0.1
capital taxation rate	Tr	0.1
Substitution elasticity	$\mu C = \mu * C = \mu I = \mu * I$	1.50
Investment adjustment costs	φX	0.50
Price Flexibility	$\xi i,$	0.75
Relative Size of the informal sector	$1 - ws$	0.5, 0.65, 0.8
Imported share of Investment	$1 - wI$	0.25
Imported share of Consumption	$1 - wC$	0.25
Share of Consumption in Exports	$Csexp$	0.1
Share of Investment in Exports	$Isexp$	0.1
standard deviation of shocks	Σi	0.01
persistence of AR1 processes	Pi	0.7

4. Discussion of Results

The result of the stochastic simulation of the model is presented in figure 1. In the model, government sector is taken as exogenous. The aggregate demand comprises of aggregate household consumption and aggregate investment, while the aggregate supply is explained by the dynamics of the aggregate output in the economy. The effect of the external sector was muted in order to focus the direct effect of the fiscal policy shock on the domestic economy.

The impulse response in figure 1 shows that aggregate output and consumption rises to about 15% in the short run. This means that expansionary fiscal policy exerts equal amount of rise in aggregate consumption and output. Similarly, the duration of equilibrium adjustment takes up to about 5 quarters for both output and consumption. This is in line with the theoretical Keynesian postulate that increases in government spending raises consumption and output in the short run. However, the result

indicates that aggregate investment falls to about 4% in the short run but adjusts back to a long-run steady state. The result shows that as aggregate private investment declines, nominal interest rate adjusts upward, suggesting rising cost of capital due to possible crowding-out of private capital investment, as government borrows to finance fiscal deficits. In addition, the result shows that the degree of informality affects short-run aggregate investment but such effect vanishes in the long run. In essence, the decline in aggregate investment in the short run is rapid and greater when informality is low (aggregate investment falls by more than 8%) than when it is high (aggregate investment falls by exactly 8%). This means formal sector investment influences aggregate investment more in the short run. The result also shows that the rise in nominal interest rate by as much as 50 basis points, implies a forward-looking monetary policy, because fiscal expansion shock is expected to be inflationary. Therefore, anticipated inflationary impact of the positive fiscal policy shock is mitigated at the onset. This forward-looking behaviour was found to have kept the short-run rise in inflation minimal and it adjusts back faster.

The result indicates that both formal and informal sector output rises in the short run, owing to the positive fiscal policy shock. However, the magnitude of the rise in formal sector output is greater with 10% compared to of informal economy output which rises by only 5%. This means that positive fiscal policy shock has greater impact on formal sector goods-producing firms' output than that of the informal economy. This also suggests that the production in the formal economy is more efficient when government spending[4] rises. Under this condition, it means that formal goods-producing firms will demand more hours of labour and other variable inputs than the informal goods-producing firms. It is worthy to note that the share of a rise in informal sector output is a loss to the revenue side of fiscal policy, largely because revenue from sales and profit tax of informal goods-producing firms leaks out of the government revenue cycle.

[4] This is an indication that most of government's domestic demand is largely supplied by the formal goods-producing firms.

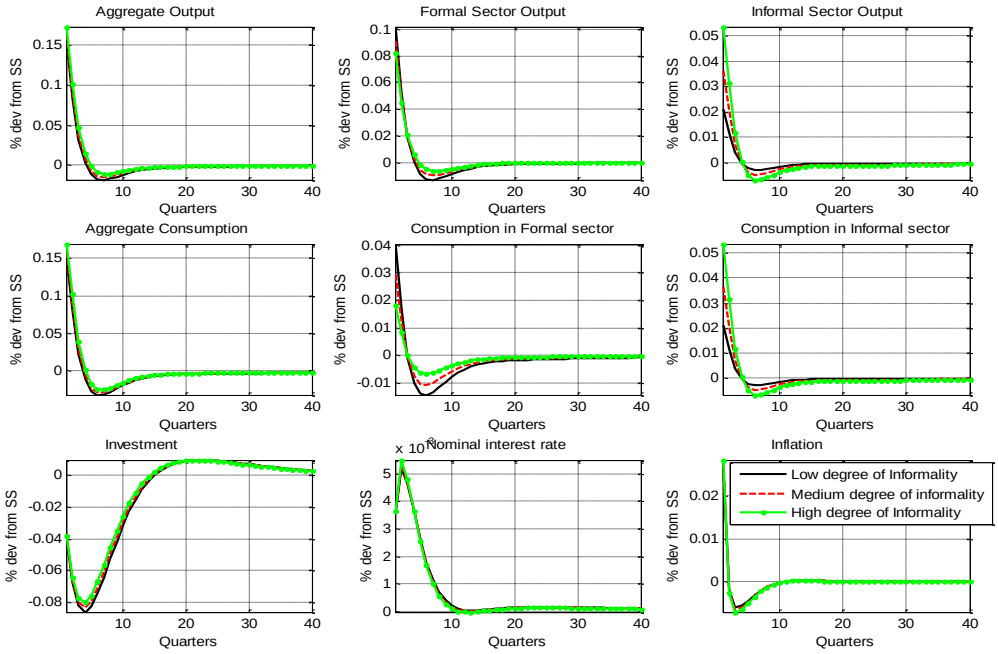

Figure 1: Fiscal Policy Shock in Small Open Oil Economy with Informality

The result also shows that the expansionary fiscal policy raised households' consumption across the two sectors. However, the informal sector good consumption increases by 5% while that of formal sector expands by only 4%. This suggests that the response of household consumption to expansionary fiscal policy differs across the two sectors. The informal sector's consumption is disproportionately larger, which implies that revenue from VAT is substantially lost in the informal sector. Thus, distorting the flow of revenue to government as informal sector goods consumption, leaks additional revenue as consumption responds to the fiscal stimulus. Therefore, as deficits rise due to expansionary fiscal policy, revenue leakage increases due to larger consumption of informal sector goods. Hence, the gap between public revenue and expenditure widens and fiscal imbalance emerges. The result implies that a larger share of the benefit of the rise in consumption due to fiscal shock is loss to informality. This means that government tax (VAT) on consumption of goods produced in the informal sector leaks out larger

revenue that should have accrued to government. Hence, the leakages from the informal economy in both the production (sales or profit tax) and consumption (VAT) side constitute a significant reason for fiscal imbalance in Nigeria. The result indicates that this imbalance has the propensity to increase as the degree of informality rises. It is also the primary cause of fiscal debt, because increasing differential between government revenue and expenditure is augmented through deficit financing (see also Ekpo, 2023). Therefore, Nigeria's debt is likely to increase as informality grows. This is also the case for most developing economies with large informal sectors. This finding is consistent with other empirical evidence such as Leal (2014), Bandaogo (2016), Dellas et al. (2017) and Ekpo (2023).

The result shows that the degree of informality has two contrasting impacts on each sector output. First is that the degree of informality does not matter when short-run output is responding to fiscal policy shock in the formal economy but it matters in equilibrium adjustment. Second is that the degree of informality matters in both size of short-run rise and equilibrium adjustment of output in the informal sector. In terms of dynamics in consumption, the higher the degree of informality, the less the rise in output in the formal sector and this implies additional leakages in government revenue.

5. Conclusion

This study sought to investigate the effect of the recent expansionary fiscal policy being pursued in the presence of a large informal sector and optimal monetary policy. The paper finds that the expansionary fiscal policy has substantially raised aggregate demand. Thus, aggregate consumption increased by as much as 15%. Consumption of goods produced in the informal sector accounts for 5% while consumption of formal sector goods makes up only 4%. Therefore, consumption of imported goods accounts for the remaining 6%. The paper argues that volatility in aggregate investment due to the expansionary fiscal policy is an indication that portfolio dominates the investment basket, for the period under review. Therefore, these investors are averse to the inflationary tendency associated with the expansionary fiscal policy. The paper observes that the aggregate output expands by 15%, with the formal sector accounting for 10% while the informal economy expands by

only 5%. Hence the formal goods-producing firms are more responsive to expansionary fiscal policy because the channel of government expenditure on domestic consumption passes directly through formal sector firms. Lastly, optimal monetary policy can complement effect of fiscal expansion by monetary policy tightening to decelerate inflationary pressure associated with the fiscal shock. Hence, forward-looking monetary authorities raise the monetary policy rate in order to tackle the inflationary effect of the fiscal expansion in the short run so that prices will be stable in the long run.

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